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| **Residuals and Influence Diagnostics** |

**Residual Diagnostics**

Consider a residual vector of the form http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0464.png, where http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0465.pngis a projection matrix, possibly an oblique projector. A typical element http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0466.pngwith variance http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0467.pngand estimated variance http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0468.pngis said to be *standardized* as

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0469.png |  |  |

and *studentized* as

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0470.png |  |  |

External studentization uses an estimate of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0471.pngthat does not involve the http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0051.pngth observation. Externally studentized residuals are often preferred over studentized residuals because they have well-known distributional properties in standard linear models for independent data.

Residuals that are scaled by the estimated variance of the response, i.e., http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0472.png, are referred to as Pearson-type residuals.

**Marginal and Conditional Residuals**

The marginal and conditional means in the linear mixed model are http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0473.pngand http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0474.png, respectively.

Accordingly, the vector http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0475.pngof marginal residuals is defined as

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0476.png |  |  |

and the vector http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0477.pngof conditional residuals is

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0478.png |  |  |

Following Gregoire, Schabenberger, and Barrett (1995), let http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0479.pngand http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0480.png. Then

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0481.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0482.png |  |  |
|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0483.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0484.png |  |  |

For an individual observation the raw, studentized, and Pearson-type residuals computed by the MIXED procedure are given in [Table 56.21](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect024.htm#statug.mixed.mixtabresidual).

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| ***Table 56.21 Residual Types Computed by the MIXED Procedure*** | | |
| **Type of Residual** | **Marginal** | **Conditional** |
| Raw | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0485.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0486.png |
| Studentized | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0487.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0488.png |
| Pearson | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0489.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0490.png |

When the [OUTPM=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodeloutpm) option is specified in addition to the [RESIDUAL](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelresidual) option in the [MODEL](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm) statement, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0491.pngand http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0492.pngare added to the data set as variables *Resid*, *StudentResid*, and *PearsonResid*, respectively.

When the [OUTP=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodeloutp) option is specified, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0493.pngand http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0494.pngare added to the data set. Raw residuals are part of the [OUTPM=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodeloutpm) and [OUTP=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodeloutp) data sets without the [RESIDUAL](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelresidual) option.

**Scaled Residuals**

For correlated data, a set of scaled quantities can be defined through the Cholesky decomposition of the variance-covariance matrix. Since fitted residuals in linear models are rank-deficient, it is customary to draw on the variance-covariance matrix of the data. If http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0495.pngand http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0496.png, then http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0497.pnghas uniform dispersion and its elements are uncorrelated.

Scaled residuals in a mixed model are meaningful for quantities based on the marginal distribution of the data.

Let http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0082.png denote the Cholesky root of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0498.png, so that http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0499.png, and define

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0500.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0501.png |  |  |
|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0502.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0503.png |  |  |

By analogy with other scalings, the inverse Cholesky decomposition can also be applied to the residual vector, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0504.png, although http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0119.pngis not the variance-covariance matrix of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0475.png.

To diagnose whether the covariance structure of the model has been specified correctly can be difficult based on http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0505.png, since the inverse Cholesky transformation affects the expected value of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0505.png. You can draw on http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0506.pngas a vector of (approximately) uncorrelated data with constant mean.

When the [OUTPM=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodeloutpm) option in the [MODEL](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm) statement is specified in addition to the [VCIRY](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelvci) option, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0505.pngis added as variable *ScaledDep* and http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0506.pngis added as *ScaledResid* to the data set.

**Influence Diagnostics**

**Basic Idea and Statistics**

The general idea of quantifying the influence of one or more observations relies on computing parameter estimates based on all data points, removing the cases in question from the data, refitting the model, and computing statistics based on the change between full-data and reduced-data estimation. Influence statistics can be coarsely grouped by the aspect of estimation that is their primary target:

* overall measures compare changes in objective functions: (restricted) likelihood distance (Cook and Weisberg 1982, Ch. 5.2)
* influence on parameter estimates: Cook’s http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0071.png(Cook 1977, 1979), MDFFITS (Belsley, Kuh, and Welsch 1980, p. 32)
* influence on precision of estimates: CovRatio and CovTrace
* influence on fitted and predicted values: PRESS residual, PRESS statistic (Allen 1974), DFFITS (Belsley, Kuh, and Welsch 1980, p. 15)
* outlier properties: internally and externally studentized residuals, leverage

For linear models for uncorrelated data, it is not necessary to refit the model after removing a data point in order to measure the impact of an observation on the model. The change in fixed effect estimates, residuals, residual sums of squares, and the variance-covariance matrix of the fixed effects can be computed based on the fit to the full data alone.

By contrast, in mixed models several important complications arise. Data points can affect not only the fixed effects but also the covariance parameter estimates on which the fixed-effects estimates depend. Furthermore, closed-form expressions for computing the change in important model quantities might not be available.

* This section provides background material for the various influence diagnostics available with the MIXED procedure.
* See the section [Mixed Models Theory](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect022.htm) for relevant expressions and definitions. The parameter vector http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0118.pngdenotes all unknown parameters in the http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0013.pngand http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0012.pngmatrix.
* The observations whose influence is being ascertained are represented by the set http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0507.pngand referred to simply as "the observations in http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0507.png."
* The estimate of a parameter vector, such as http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0003.png, obtained from all observations except those in the set http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0507.pngis denoted http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0508.png. In case of a matrix http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0127.png, the notation http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0509.pngrepresents the matrix with the rows in http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0507.pngremoved; these rows are collected in http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0510.png. If http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0127.pngis symmetric, then notation http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0509.pngimplies removal of rows and columns.
* The vector http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0511.pngcomprises the responses of the data points being removed, and http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0512.pngis the variance-covariance matrix of the remaining observations. When http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0513.png, lowercase notation emphasizes that single points are removed, such as http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0514.png.

**Managing the Covariance Parameters**

An important component of influence diagnostics in the mixed model is the estimated variance-covariance matrix http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0515.png. To make the dependence on the vector of covariance parameters explicit, write it as http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0516.png. If one parameter, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0007.png, is profiled or factored out of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0119.png, the remaining parameters are denoted as http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0517.png. Notice that in a model where http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0012.pngis diagonal and http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0015.png, the parameter vector http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0517.pngcontains the ratios of each variance component and http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0007.png(see Wolfinger, Tobias, and Sall 1994). When [ITER=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluenceiter)0, two scenarios are distinguished:

1. If the residual variance is not profiled, either because the model does not contain a residual variance or because it is part of the Newton-Raphson iterations, then http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0518.png.
2. If the residual variance is profiled, then http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0519.pngand http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0520.png. Influence statistics such as Cook’s http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0071.pngand internally studentized residuals are based on http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0521.png, whereas externally studentized residuals and the DFFITS statistic are based on http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0522.png. In a random components model with uncorrelated errors, for example, the computation of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0523.pnginvolves scaling of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0401.pngand http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0402.pngby the full-data estimate http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0524.pngand multiplying the result with the reduced-data estimate http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0525.png.

Certain statistics, such as MDFFITS, CovRatio, and CovTrace, require an estimate of the variance of the fixed effects that is based on the reduced number of observations. For example, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0523.pngis evaluated at the reduced-data parameter estimates but computed for the entire data set. The matrix http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0526.png, on the other hand, has rows and columns corresponding to the points in http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0507.pngremoved. The resulting matrix is evaluated at the delete-case estimates.

When influence analysis is iterative, the entire vector http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0118.pngis updated, whether the residual variance is profiled or not. The matrices to be distinguished here are http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0527.png, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0528.png, and http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0526.png, with unambiguous notation.

**Predicted Values, PRESS Residual, and PRESS Statistic**

An unconditional predicted value is http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0529.png, where the vector http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0530.pngis the http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0051.pngth row of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0004.png. The (raw) residual is given as http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0531.png, and the PRESS *residual* is

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0532.png |  |  |

The PRESS *statistic* is the sum of the squared PRESS residuals,

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0533.png |  |  |

where the sum is over the observations in http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0507.png.

If [EFFECT=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluenceeffect), [SIZE=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluencesize), or [KEEP=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluencekeep) is not specified, PROC MIXED computes the PRESS residual for each observation selected through [SELECT=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluenceselect) (or all observations if [SELECT=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluenceselect) is not given). If [EFFECT=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluenceeffect), [SIZE=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluencesize), or [KEEP=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluencekeep) is specified, the procedure computes http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0534.png.

**Leverage**

For the general mixed model, leverage can be defined through the projection matrix that results from a transformation of the model with the inverse of the Cholesky decomposition of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0119.png, or through an oblique projector. The MIXED procedure follows the latter path in the computation of influence diagnostics. The leverage value reported for the http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0051.pngth observation is the http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0051.pngth diagonal entry of the matrix

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0535.png |  |  |

which is the weight of the observation in contributing to its own predicted value, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0536.png.

While http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0537.pngis idempotent, it is generally not symmetric and thus not a projection matrix in the narrow sense.

The properties of these leverages are generalizations of the properties in models with diagonal variance-covariance matrices. For example, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0538.png, and in a model with intercept and http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0539.png, the leverage values

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0540.png |  |  |

are http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0541.pngand http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0542.png. The lower bound for http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0543.pngis achieved in an intercept-only model, and the upper bound is achieved in a saturated model. The trace of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0537.pngequals the rank of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0004.png.

If http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0544.pngdenotes the element in row http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0051.png, column http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0042.pngof http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0545.png, then for a model containing only an intercept the diagonal elements of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0537.pngare

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| --- | --- | --- | --- |
|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0546.png |  |  |

Because http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0547.pngis a sum of elements in the http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0051.pngth row of the ***inverse*** variance-covariance matrix, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0543.pngcan be negative, even if the correlations among data points are nonnegative. In case of a saturated model with http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0548.png, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0549.png.

**Internally and Externally Studentized Residuals**

See the section [Residual Diagnostics](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect024.htm#statug.mixed.mixeddetailres) for the distinction between standardization, studentization, and scaling of residuals. Internally studentized marginal and conditional residuals are computed with the [RESIDUAL](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelresidual) option of the [MODEL](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm) statement. The [INFLUENCE](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluence) option computes internally and externally studentized marginal residuals.

The computation of internally studentized residuals relies on the diagonal entries of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0550.png, where http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0551.png. Externally studentized residuals require iterative influence analysis or a profiled residual variance. In the former case the studentization is based on http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0523.png; in the latter case it is based on http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0552.png.

**Cook’s http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0071.png**

Cook’s http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0071.pngstatistic is an invariant norm that measures the influence of observations in http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0507.pngon a vector of parameter estimates (Cook 1977). In case of the fixed-effects coefficients, let

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0553.png |  |  |

Then the MIXED procedure computes

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0554.png |  |  |

where http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0555.pngis the matrix that results from sweeping http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0556.png.

If http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0119.pngis known, Cook’s http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0071.pngcan be calibrated according to a chi-square distribution with degrees of freedom equal to the rank of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0004.png(Christensen, Pearson, and Johnson 1992). For estimated http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0119.pngthe calibration can be carried out according to an http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0557.pngdistribution. To interpret http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0071.pngon a familiar scale, Cook (1979) and Cook and Weisberg (1982, p. 116) refer to the 50th percentile of the reference distribution. If http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0071.pngis equal to that percentile, then removing the points in http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0507.pngmoves the fixed-effects coefficient vector from the center of the confidence region to the 50% confidence ellipsoid (Myers 1990, p. 262).

In the case of iterative influence analysis, the MIXED procedure also computes a http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0071.png-type statistic for the covariance parameters. If http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0558.pngis the asymptotic variance-covariance matrix of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0122.png, then MIXED computes

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0559.png |  |  |

**DFFITS and MDFFITS**

A DFFIT measures the change in predicted values due to removal of data points. If this change is standardized by the externally estimated standard error of the predicted value in the full data, the DFFITS statistic of Belsley, Kuh, and Welsch (1980, p. 15) results:

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0560.png |  |  |

The MIXED procedure computes DFFITS when the [EFFECT=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluenceeffect) or [SIZE=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluencesize) modifier of the [INFLUENCE](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluence) option is not in effect. In general, an external estimate of the estimated standard error is used. When [ITER](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluenceiter) > 0, the estimate is

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0561.png |  |  |

When [ITER=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluenceiter)0 and http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0007.pngis profiled, then

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0562.png |  |

When the [EFFECT=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluenceeffect), [SIZE=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluencesize), or [KEEP=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluencekeep) modifier is specified, the MIXED procedure computes a multivariate version suitable for the deletion of multiple data points. The statistic, termed MDFFITS after the MDFFIT statistic of Belsley, Kuh, and Welsch (1980, p. 32), is closely related to Cook’s http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0071.png. Consider the case http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0563.pngso that

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0564.png |  |  |

and let http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0565.pngbe an estimate of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0566.pngthat does not use the observations in http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0507.png. The MDFFITS statistic is then computed as

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0567.png |  |  |

If [ITER=](http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/statug_mixed_sect015.htm#statug.mixed.mixedmodelinfluenceiter)0 and http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0007.pngis profiled, then http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0568.pngis obtained by sweeping

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0569.png |  |  |

The underlying idea is that if http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0517.pngwere known, then

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0570.png |  |  |

would be http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0571.pngin a generalized least squares regression with all but the data in http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0507.png.

In the case of iterative influence analysis, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0565.pngis evaluated at http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0572.png. Furthermore, a MDFFITS-type statistic is then computed for the covariance parameters:

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0573.png |  |  |

**Covariance Ratio and Trace**

These statistics depend on the availability of an external estimate of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0119.png, or at least of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0007.png. Whereas Cook’s http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0071.pngand MDFFITS measure the impact of data points on a vector of parameter estimates, the covariance-based statistics measure impact on their precision. Following Christensen, Pearson, and Johnson (1992), the MIXED procedure computes

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0574.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0575.png |  |  |
|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0576.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0577.png |  |  |

where http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0578.pngdenotes the determinant of the nonsingular part of matrix http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0579.png.

In the case of iterative influence analysis these statistics are also computed for the covariance parameter estimates. If http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0063.pngdenotes the rank of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0580.png, then

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0581.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0582.png |  |  |
|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0583.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0584.png |  |  |

In the case of iterative influence analysis, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0565.pngis evaluated at http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0572.png. Furthermore, a MDFFITS-type statistic is then computed for the covariance parameters:

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0573.png |  |  |

**Covariance Ratio and Trace**

These statistics depend on the availability of an external estimate of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0119.png, or at least of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0007.png. Whereas Cook’s http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0071.pngand MDFFITS measure the impact of data points on a vector of parameter estimates, the covariance-based statistics measure impact on their precision. Following Christensen, Pearson, and Johnson (1992), the MIXED procedure computes

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0574.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0575.png |  |  |
|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0576.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0577.png |  |  |

where http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0578.pngdenotes the determinant of the nonsingular part of matrix http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0579.png.

In the case of iterative influence analysis these statistics are also computed for the covariance parameter estimates. If http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0063.pngdenotes the rank of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0580.png, then

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0581.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0582.png |  |  |
|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0583.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0584.png |  |  |

In the case of iterative influence analysis, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0565.pngis evaluated at http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0572.png. Furthermore, a MDFFITS-type statistic is then computed for the covariance parameters:

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0573.png |  |  |

**Covariance Ratio and Trace**

These statistics depend on the availability of an external estimate of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0119.png, or at least of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0007.png. Whereas Cook’s http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0071.pngand MDFFITS measure the impact of data points on a vector of parameter estimates, the covariance-based statistics measure impact on their precision. Following Christensen, Pearson, and Johnson (1992), the MIXED procedure computes

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0574.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0575.png |  |  |
|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0576.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0577.png |  |  |

where http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0578.pngdenotes the determinant of the nonsingular part of matrix http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0579.png.

In the case of iterative influence analysis these statistics are also computed for the covariance parameter estimates. If http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0063.pngdenotes the rank of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0580.png, then

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0581.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0582.png |  |  |
|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0583.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0584.png |  |  |

In the case of iterative influence analysis, http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0565.pngis evaluated at http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0572.png. Furthermore, a MDFFITS-type statistic is then computed for the covariance parameters:

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0573.png |  |  |

**Covariance Ratio and Trace**

These statistics depend on the availability of an external estimate of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0119.png, or at least of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0007.png. Whereas Cook’s http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0071.pngand MDFFITS measure the impact of data points on a vector of parameter estimates, the covariance-based statistics measure impact on their precision. Following Christensen, Pearson, and Johnson (1992), the MIXED procedure computes

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| --- | --- | --- | --- | --- |
|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0574.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0575.png |  |  |
|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0576.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0577.png |  |  |

where http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0578.pngdenotes the determinant of the nonsingular part of matrix http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0579.png.

In the case of iterative influence analysis these statistics are also computed for the covariance parameter estimates. If http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0063.pngdenotes the rank of http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0580.png, then

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|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0581.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0582.png |  |  |
|  | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0583.png | http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/images/statug_mixed0584.png |  |  |